

Exploring tactical choices and game design outcomes in a simple wargame ‘Take that Hill’ by a systematic approach using Experimental Design

Mark Flanagan¹, Adrian Northey², Ian M Robinson²
¹NHS Business Service Authority, Newcastle upon Tyne, UK.
²Hearts of Oak, North Yorks, UK, a.anser4u@gmail.com

Abstract

Experimental Design (ED) technique is a proven analytical method used in the chemicals industry. We have taken this approach and applied it to Phil Sabin's ‘Take That Hill’, a simple wargame presented at Connections 2014. By evolving the tactical turn game choices into playable full-game strategies, a descriptive set of game outcomes can be delivered and optimised to produce winning strategies. This provides a systematic approach to testing a game, with full post-game deconstructive analysis which is capable of being used to identify flaws, and find optimal strategies in playing the game. The most successful strategies found by ED outperformed individual strategies developed by experienced players. ED allowed pairing of obvious good play with seemingly counterintuitive play that were found to work well in unexpected combinations.

Keywords: Wargames, Experimental Design, Strategies, Verification and Validation;

1 Introduction

‘Take That Hill’ is a simple wargame that explores the concepts of fire and movement. It was presented at the 2014 UK Connections conference at King's College, London by its creator Professor Phil Sabin [1]. It was adapted by the Royal Military Academy Sandhurst as a simple teaching tool [2]. The aim of the wargame is to experiment and discuss different methods of fire and movement at the platoon level. There are two players denoted Blue and Red, with Blue attacking and Red defending a position on a hill. The Blue Player commands an Infantry Platoon made up of three sections. The Platoon Commander (COMD) is also represented and must move with one of the sections at all times. COMD may move between sections that are adjacent to each other, provided they have not already moved that turn. The Red player commands the opposing force of an enemy section. The Blue sections are tasked to advance across open ground with no cover from view or fire. About 500m away is a small hill on which the Red enemy section has hastily dug in. The Blue sections must clear the Red section off the hill, by advancing into their hex, whereas Red's aim is to delay Blue for as long as possible and attrite them in the process. Our study of this game has two motives. Firstly through the use of Experimental Design, the game ‘Take That Hill’ was systematically tested to find out if it can be applied to rule based games. Secondly, winning strategies and tradeoffs in the game can be quickly identified in an unbiased manner with a view to improving tactics in practice, which is an essential part of military training. Our study demonstrates Experimental Design methodology is generally applicable for robust games testing, and can identify likely winning strategies, which are sometimes surprising. This has a utility to the wider games design community.



The map used in ‘Take That Hill’ is shown in Figure 1 [2] and the game is described in detail below.

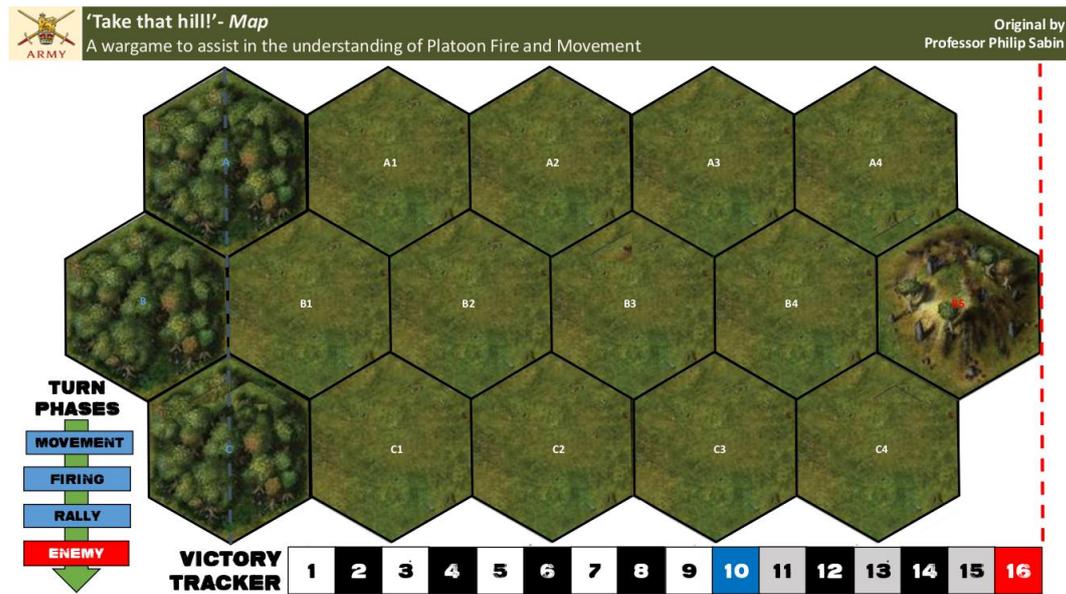


Figure 1: ‘Take that Hill’ map.

The game is played with four phases per turn; 3 Blue, 1 Red. After all phases have been played the turn ends.

1. Movement. Each fresh section may ‘move’ to any adjacent hex and become spent. A fresh section that remains still in its current hex does not become spent. A spent section may not move. The Platoon Commander may move from a fresh section to any adjacent section even if this section is spent.

2. Firing. Each fresh section may fire to suppress the enemy if desired, doing so turns the section spent. A spent section may not fire. To fire, roll a dice, and if the number exceeds the range in hexes from the firer to the hill, the enemy section becomes spent. If the roll is equal to, or less than the range, the fire does not suppress. Because the enemy is situated on a hill there are no line of fire restrictions on friendly sections firing ‘through’ one another.

3. Rally. Any sections that are in the same hex as COMD are rallied automatically and become fresh. Any other spent sections must roll a dice and will become fresh if the roll exceeds the distance in hexes to the COMD hex, if the roll is equal to or lower the section remains spent.

4. Enemy Action. If the enemy section is spent when this phase commences it automatically becomes fresh and then the phase ends. If it is already fresh it may fire on any of the hexes containing a Blue section as the Red Player wishes. If there is another section in an adjacent hex to the ‘target hex’ this too may be fired upon in the open. Any sections that are in the open are engaged. For each section that is engaged the Red player rolls a dice, aiming to roll equal to or more than the distance to the target hexes in question. Successful rolls result in ‘hits’ on the Blue sections turning them spent if not already so. Every ‘hit’ moves the Victory Counter along one on the Victory Tracker. The COMD is able to be independently targeted and does not count [3] as an additional hit if the section they are with is successfully engaged. The enemy section always finishes the turn fresh.

In the game each hit on Blue, Red and the time to reach the hill is recorded. Figure 1 suggests that a successful attack occurs if the hill is reached with 10 or less hits on Blue, a draw occurs if the hits on Blue is between 11 to 15, and a loss if 16 or more hits are suffered by Blue regardless of the hits made on Red, or the time to reach the hill, as the price paid is deemed too great.

2 Investigating the game systematically

This simple game lends itself to investigation by a number of methods. Firstly it can be approached from a procedural/process orientated direction, identifying data structures that describe quantifiable aspects of its micro-world and the processes that would work in them. This is related to the notion of a wargaming schema definition as described by Dr Peter Perla [4] with the assumption being that the game would iterate to a finite endpoint with Blue developing strategies through repeatable legal moves of game pieces. If the section isn't spent at the start of the game turn (which constrains it to neither move or fire until rallied in the game), the game move schema in Table 1 gives a rich option of moves.

Table 1: Individual game move schema for 'Take that Hill' before considerations of being spent come into play. The numbers represent each section, with the letters F & M being fire or move respectively. Each option (a-h) is unique.

Option	Section 1	Section 2	Section 3
a	F	F	F
b	F	F	M
c	F	M	F
d	M	F	F
e	F	M	M
f	M	F	M
g	M	M	F
h	M	M	M

Option (a) would involve sections 1-3 firing but not moving, option (h) would involve sections 1-3 moving but not firing, options (b, c & d) involve two sections firing and one moving, etc.

In addition, three other variables must be considered. These are move positions (from the front or the back), section dispersion (concentrated or dispersed) and COMD position (either leading from the front or the rear). These are discussed at length in Appendix 1 together with figures explaining the differences representing the range of choices available in the game.

A systematic approach to these choices is by Experimental Design which is an intrinsically parallel investigation of multiple variables, allowing optimal solutions to be found. It is widely applied in the chemical and other processing industries for both new product formulation [5] and plant/process optimisation [6]. Full factorial Experimental Design allows the study of the effects that several factors can have on a response at all combinations of the factor levels. The number of runs for n factors under investigation is 2^n and an average effect is determined for each factor by multiple regression analysis such as best subsets, with each variable selected in the regression model by significance tests in an unbiased way [7]. Experiments are run at different factor values, called levels. To better understand variability if time or resource permits it is conventional to repeat one of the runs a few times. There is a limit in utility to repeatedly measuring the same run, as each variable is already measured 2^n times by virtue of the design, and since the confidence interval for the variable depends upon $1/\sqrt{\text{number of runs}}$, a large number of repeated runs is less important than one might think. The method identifies the variables with the strongest average effect selected by significance tests which removes weaker effects from the final regression models.

In game terms we have used Experimental Design to discover and apply winning strategies by selecting a subset of the possible moves outlined above (b, c, d & h) and exploring these in relation to other move options using the method below.

A 2⁴ factorial Experimental Design (see page 274, example 7.2 and Table M in reference [6] for details) was used to generate the matrix, resulting in 16 runs, each with distinct tactical choices using four separate variables described in the paper, and they are shown schematically in Appendix 1. The +1 level indicates it would increase the responses under study, and conversely the -1 level indicates it would decrease the responses under study.

Assignment

Variable A – Move sections	
Move 1 section if possible, the remaining section fire (options b-d).	-1
Move as many as possible (option h).	+1
Variable B – Move position	
Always move from the back.	-1
Always move from the front.	+1
Variable C – Disperse	
No (keep as a solid block).	-1
Yes (spread out).	+1
Variable D – COMD	
Platoon command leads from the back to rally stragglers.	-1
Platoon command leads from the front to lead by example.	+1

Each of these variables is shown graphically in Figures 1-4 in Appendix 1 to illustrate the patterns used.

Other variables in movement (such as (a) in Table 1) where all sections fire but none move were discounted as the game requires movement to reach the victory conditions. If the game included cover between the woods and the hill, then this would have been incorporated into the design, and consequently the variables chosen in this study reflect just this game alone, with more complex games requiring more variables. To support the game play, a python based Computer Assisted Instructional (CAI) program [8] was written (in Python 3) to help the researchers iterate through the phased sequence of play in ‘Take That Hill’, so that the many game iterations required for the experiment were played out faithfully and consistently. This meant that the rule-set (as in the “actions and consequences”) were correctly applied for each game run. Inside the ‘Take That Hill’ Python program, local data structures were used to encapsulate the board states and verify that only allowed (legal game play) moves were accepted, allowing the manual dice rolling and the analogue game board to progress. The experimenters were thus prompted for actions at the appropriate times and the data automatically recorded for verification and play-back if necessary (alongside photographs taken of each game turn, used for verification).

```
python3.8
This is Take That Hill

What are the Blue Starting Positions
Where is Section 1?[A, B or C]1
Where is Section 2?[A, B or C]2
Where is Section 3?[A, B or C]3
Which section is the leader with? [1, 2, or 3]2

Blue Phase: Turn 1

Turn 1

Row A:   ....A .... .... .... ....
Row B:   ....B ..... .... .... .XX.
Row C:   ....C ..... .... .... ....

Section 1 is Active being located in hex: 1
Section 2 is Active being located in hex: 2 with the Platoon Leader
Section 3 is Active being located in hex: 3

Blue Move
Section 1 in 1 has Legal Moves of: ['N']
Do you want to move Section 1 [Hex or N]?
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Figure 2: 'Take that Hill' Computer Assisted Instructional (CAI) program.

The Experimental Design gives the following matrix of variables that are combined into a consistent choice of tactics for one run through the game.

Table 2: 2^4 factorial Experimental Design applied to 'Take that Hill' before considerations of being spent come into play.

						A	B	C	D
Run	Pattern	A	B	C	D	Move sections	Move position	Disperse	COMD
1	1	-1	-1	-1	-1	1 if possible	From back	No	Back
2	a	+1	-1	-1	-1	3 if possible	From back	No	Back
3	b	-1	+1	-1	-1	1 if possible	From front	No	Back
4	ab	+1	+1	-1	-1	3 if possible	From front	No	Back
5	c	-1	-1	+1	-1	1 if possible	From back	Yes	Back
6	ac	+1	-1	+1	-1	3 if possible	From back	Yes	Back
7	bc	-1	+1	+1	-1	1 if possible	From front	Yes	Back
8	abc	+1	+1	+1	-1	3 if possible	From front	Yes	Back
9	d	-1	-1	-1	+1	1 if possible	From back	No	Front
10	ad	+1	-1	-1	+1	3 if possible	From back	No	Front
11	bd	-1	+1	-1	+1	1 if possible	From front	No	Front
12	abd	+1	+1	-1	+1	3 if possible	From front	No	Front
13	cd	-1	-1	+1	+1	1 if possible	From back	Yes	Front
14	acd	+1	-1	+1	+1	3 if possible	From back	Yes	Front
15	bcd	-1	+1	+1	+1	1 if possible	From front	Yes	Front
16	abcd	+1	+1	+1	+1	3 if possible	From front	Yes	Front

The first column indicated the run number in the game matrix. The lower case letters in the second column a, b, ab etc. indicate runs which have variables set to the +1 value; the number 1 indicates that all variables are set to the -1 value. The assignments +1 or -1 in columns 3 to 6 correspond to the choices defined above for variables A-D, and columns 7-10 describe each variable with words. Thus run 1 comprises simultaneously moving one section and firing two if possible, ensuring this section is the one furthest at the back, keeping together as a solid block of sections rather than dispersing, and finally making the platoon command (COMD) lead from the back to rally stragglers. The following examples demonstrate graphically runs 1 and 10 from the matrix in Table 2.

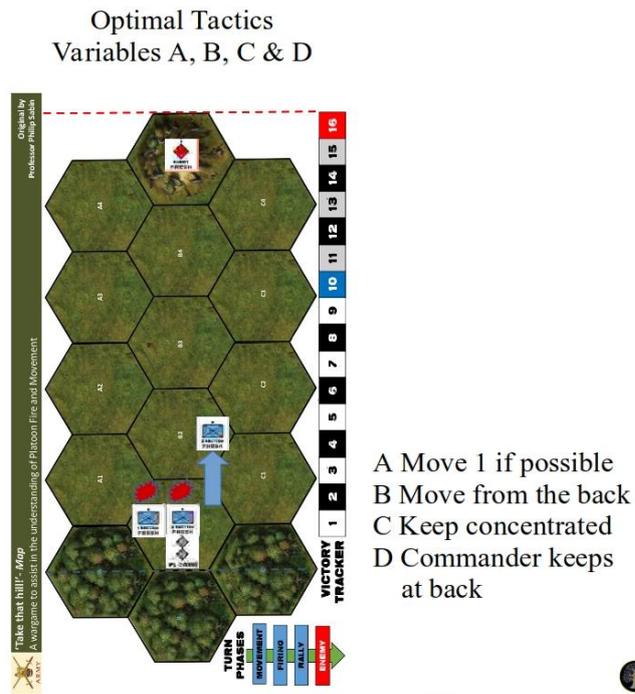


Figure 3. Example of the details of Run 1.

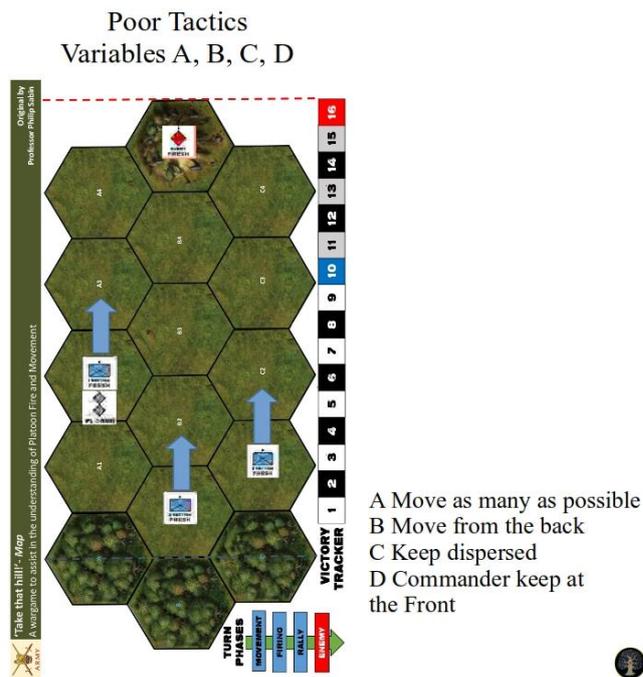


Figure 4: Example of the details of Run 10.

If a Blue section is capable of closing down on the Red section on the hill by being adjacent to it, it does so, regardless of the run move sequence suggested by Table 2 to end the game. At the end, the total number of hits on Red, Blue and the moves to take the hill are recorded.

3 Results and analysis

The outcome of each run was measured in terms of the number of casualties sustained by both the Red and Blue sides and the number of moves required to take the hill held by the Red group. The results are shown below.

Table 3: Results from 2⁴ factorial Experimental Design applied to 'Take that Hill'.

	A	B	C	D	Red Hits	Blue Hits	Time to take hill
Run	Move sections	Move position	Disperse	COMD			
1	1 if possible	From back	No	Back	14	1	12
2	3 if possible	From back	No	Back	0	25	7
3	1 if possible	From front	No	Back	2	18	11
4	3 if possible	From front	No	Back	0	22	11
5	1 if possible	From back	Yes	Back	8	6	13
6	3 if possible	From back	Yes	Back	0	11	5
7	1 if possible	From front	Yes	Back	2	6	9
8	3 if possible	From front	Yes	Back	0	18	12
9	1 if possible	From back	No	Front	13	5	13
10	3 if possible	From back	No	Front	0	18	10
11	1 if possible	From front	No	Front	2	8	11
12	3 if possible	From front	No	Front	0	8	6
13	1 if possible	From back	Yes	Front	8	5	11
14	3 if possible	From back	Yes	Front	0	15	9
15	1 if possible	From front	Yes	Front	1	7	7
16	3 if possible	From front	Yes	Front	0	10	10

The runs were then analysed using multiple regression methods [7] resulting in a detailed understanding of trade-offs within the game described in Appendix 2 for each metric. To explore this further we'll look at optimum strategy for each metric.

The final regression model for Red hits was

$$\text{Red hits} = 3.13 - 3.12 A - 2.25 B - 0.75 C \quad (1)$$

In the case of Red hits, there are negative correlations to all variables and this can be best achieved by using runs with the negative values for variables A-C.

Variable A – Move sections

Move 1 section if possible, the remaining section fire (options b-d).	-1
Variable B – Move position	
Always move from the back.	-1
Variable C – Disperse	
No (keep as a solid block).	-1

should rank highest. Ordering the data in Table 3 from highest Red hits to lowest Red hits gives:

Table 4: Results from 2⁴ factorial Experimental Design applied to ‘Take that Hill’ ordered by Red Hits.

	A	B	C	D	Red Hits	Blue Hits	Time to take hill
Run	Move sections	Move position	Disperse	COMD			
1	1 if possible	From back	No	Back	14	1	12
9	1 if possible	From back	No	Front	13	5	13
13	1 if possible	From back	Yes	Front	8	5	11
5	1 if possible	From back	Yes	Back	8	6	13
7	1 if possible	From front	Yes	Back	2	6	9
11	1 if possible	From front	No	Front	2	8	11
3	1 if possible	From front	No	Back	2	18	11
15	1 if possible	From front	Yes	Front	1	7	7
12	3 if possible	From front	No	Front	0	8	6
16	3 if possible	From front	Yes	Front	0	10	10
6	3 if possible	From back	Yes	Back	0	11	5
14	3 if possible	From back	Yes	Front	0	15	9
8	3 if possible	From front	Yes	Back	0	18	12
10	3 if possible	From back	No	Front	0	18	10
4	3 if possible	From front	No	Back	0	22	11
2	3 if possible	From back	No	Back	0	25	7

One can see the optimal strategy highlighted in the regression equation of moving 1 section if possible, the remaining sections fire, always moving from the back and not dispersing, but keeping as a solid block appears in the top two in the table. It seems contrary to common military sense to keep concentrated rather than dispersed when attacking a position in the open. We believe this outcome is due to the rule which confers automatic rallying by units in the same hex as COMD and is thus an inadvertent consequence of the game design.

In the case of Blue hits, there is a positive correlation to A and a negative correlation to C & D, so minimising Blue hits would require choosing the negative values for A, and the positive value for C & D.

$$\text{Blue hits} = 11.44 + 4.44 A - 1.69 C - 1.94 D \quad (2)$$

- Variable A – Move sections
 - Move 1 section if possible, the remaining section fire (options b-d). -1
- Variable C – Disperse
 - Yes (spread out). +1
- Variable D – COMD
 - Platoon command leads from the front to lead by example. +1

Ordering the data in Table 3 from lowest Blue hits to highest Blue hits gives:

Table 5: Results from 2⁴ factorial Experimental Design applied to ‘Take that Hill’ reverse ordered by Blue Hits.

	A	B	C	D	Red Hits	Blue Hits	Time to take hill
Run	Move sections	Move position	Disperse	COMD			
1	1 if possible	From back	No	Back	14	1	12
9	1 if possible	From back	No	Front	13	5	13
13	1 if possible	From back	Yes	Front	8	5	11
5	1 if possible	From back	Yes	Back	8	6	13
7	1 if possible	From front	Yes	Back	2	6	9
15	1 if possible	From front	Yes	Front	1	7	7
11	1 if possible	From front	No	Front	2	8	11
12	3 if possible	From front	No	Front	0	8	6
16	3 if possible	From front	Yes	Front	0	10	10
6	3 if possible	From back	Yes	Back	0	11	5
14	3 if possible	From back	Yes	Front	0	15	9
3	1 if possible	From front	No	Back	2	18	11
8	3 if possible	From front	Yes	Back	0	18	12
10	3 if possible	From back	No	Front	0	18	10
4	3 if possible	From front	No	Back	0	22	11
2	3 if possible	From back	No	Back	0	25	7

Again one can see the optimal strategy highlighted in the regression equation of moving 1 section if possible, the remaining sections fire, always moving from the back and not dispersing, but keeping as a solid block appears in the top two in the table. It is likely that the outcome when keeping concentrated rather than dispersed is due to the rule which confers automatic rallying by units in the same hex as COMD, and is thus a consequence of game design. The top eight runs show a clear bias towards moving 1 section if possible, and slightly to the platoon command leading from the front, with variables B and C split evenly.

At this stage, runs 1, 9 & 13 look optimal for taking the position based on minimum Blue and maximum Red casualties, with run 13 slightly fewer moves to take the hill.

In the case of time to take the hill, there is a negative correlation to A.

$$\text{Time to take hill} = 9.81 - 1.063 A \quad (3)$$

Variable A – Move sections

Move as many as possible +1

This is a bayonet charge, and a completely different strategy to those identified above. Ordering the data in Table 3 from lowest to highest time to take the hill, then by lowest to highest Blue casualties gives:

Table 6: Results from 2⁴ factorial Experimental Design applied to 'Take that Hill' reverse ordered by time to take the hill, then reverse ordered by Blue Hits.

	A	B	C	D	Red Hits	Blue Hits	Time to take hill
Run	Move sections	Move position	Disperse	COMD			
6	3 if possible	From back	Yes	Back	0	11	5
12	3 if possible	From front	No	Front	0	8	6
15	1 if possible	From front	Yes	Front	1	7	7
2	3 if possible	From back	No	Back	0	25	7
7	1 if possible	From front	Yes	Back	2	6	9
14	3 if possible	From back	Yes	Front	0	15	9
16	3 if possible	From front	Yes	Front	0	10	10
10	3 if possible	From back	No	Front	0	18	10
13	1 if possible	From back	Yes	Front	8	5	11
11	1 if possible	From front	No	Front	2	8	11
3	1 if possible	From front	No	Back	2	18	11
4	3 if possible	From front	No	Back	0	22	11
1	1 if possible	From back	No	Back	14	1	12
8	3 if possible	From front	Yes	Back	0	18	12

9	1 if possible	From back	No	Front	13	5	13
5	1 if possible	From back	Yes	Back	8	6	13

One can see the optimal strategy highlighted in the regression equation of moving all 3 sections in a bayonet charge appears in the top two in the table. Run 15 is the shortest time to take the hill with minimum Blue casualties. The top eight runs show a clear bias towards moving all 3 sections in a bayonet charge, and slightly to keeping dispersed and COMD leading from the front, with variable B split evenly. In order to check repeatability, runs 1 and 13 were repeated a further four times each, with the data shown in Appendix 3, together with the mean and standard deviation. In all cases the data were found to conform to the gaussian distribution, allowing for estimation of ranges based on probability. The data indicated advantages for run 1 compared to run 13, confirming the understanding of the role of the platoon commander in the game, and as a result, counterintuitively the benefits of keeping concentrated rather than dispersed.

4 Conclusions

The tactical choices in the Experimental Design revealed some strategies that were highly successful, whereas others were extremely poor. Run 1 is indeed the optimal run to use for minimising the number of Blue casualties. In terms of taking the hill in the shortest number of moves, regardless of casualties, then runs 6 and 12 look appropriate. The optimum strategy, run 1, was quite unexpected and seems counterintuitive to real life expectations and may reflect an unrealistic aspect of the game's design. Experienced wargamers tried their own individual strategies but did not manage to beat the optimum, run 1.

No research is ever truly complete, as no game design is ever finished. The 'Take That Hill' model was deconstructed from a first viewing by play testing and performing diagnostic run throughs to fully understand the axiomatic structure of the game. Though not from a military background the researchers used Connections UK 2013-2019 Conferences to provide the researchers first-hand contact with military professionals and experience deconstructing manual military game systems. The key principles of concentration/dispersion in command and control, alongside the principle of Fire and Movement came from this and led to the refinement of the four Experimental Design parameters. This subset of all possible parameters was explored in the discussed research. Further research is needed to explore "incomplete information scenarios" (the Fog of War) and the introduction of time-critical dependencies, where speed is of the essence.

There is continuous modification, improvement and reassessment which can even lead to an expansion or complete restructuring of the schema of a micro-world. The tally of manual games played by us to date is well over fifty, so we have a distinct feel of how 'Take That Hill' plays over a broad range of playing strategies and we can highlight the most successful variations, thanks to the Experimental Design technique; It is possible to play any desired number of iterations with a suitably coded version of the game. To do that the step change required is from a manual game, to computer assisted and partially automated and then to a fully automated schema. This will be part of our next cycle of research.

The research findings reported show how the Computer Assisted Instruction (CAI) assisted the replicability and confidence in consistently applying Experimental Design methodology in application to a manual game model. This was shown to be analogous to the high stakes commercial implementation processes using Experimental Design in the chemical industry. Further, the CAI approach allowed verification that a future closed-loop system was viable, by opening-up results to inspection and building confidence in this next

step. The result of this phase of research was a model ready for full automation, verified by professional Wargame designers undertaking the study. However computationally expensive the construction of a micro-world seems to be, if based on a well-tested analogue format game aka a play-tested manual game [9] the ‘time to market’ or ‘development cycle’ is greatly reduced but more importantly it paves the way for the Experimental Design technique to take place quickly. It marries nicely with AGILE, SCRUM, Sprint based software development methodology [10,11] which leads to a Minimal Viable Product (MVP) that has “immediate worth” and is delivered quickly. A fully automated schema (as in a closed loop as opposed to open loop control) would increase the possible number of game runs to be generated (no longer needing the manual analogue to be run alongside as verification), with the ability to take this frequency to any desired number of iterations. Another ‘not to be underestimated’ advantage is that the generated game-play feedback can be presented visually, events are open to inspection and over “many” game-runs, so low probability outcomes can be explored, which are anticipated from the measured gaussian distribution for each game outcome. The narrative feedback to the players should intuitively ring true or be challenged, the characteristics of actual terrain should always trump the iconised map symbols, be it from Aldershot memories of attacking Cheese Hill or other hard-earned knowledge. Experimental Design is well suited for continuous evaluation and data-to-reality mismatch investigation - to tease out unrealities akin to those that are seen in Computer Game Design [12,13].

Experimental Design should therefore be viewed as another quantitative tool in the game designers and analyst’s arsenal, but one in which its results can also be expressed in qualitative terms, the game narrative of experiential role-play [14, 15]. By game re-play, a successful evolved strategy can be viewed alongside alternative other possible strategies and evaluated; discontinuities with perceived reality should lead to focused investigation which should lead to better wargaming models. In a robustly tested wargame used for real planning, the Experimental Design process offers opportunity for identifying optimal strategies in a systematic manner as outlined in this paper.

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Appendix 1 *Graphical representations of variables A-D*

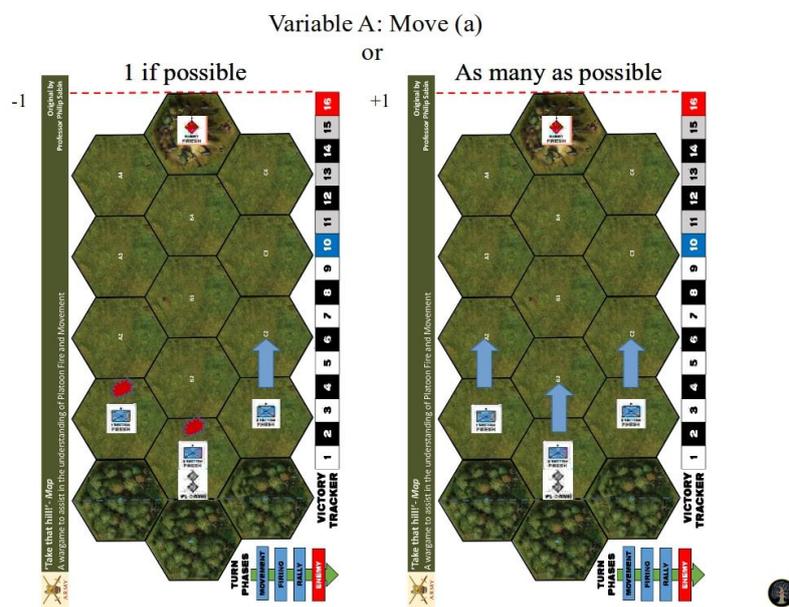


Figure 5: Variable A in the Experimental Design. The difference between moving 1 section, with the rest firing -1 , or all 3 sections moving +1.

Comments: Moving 1 section and firing 2 gives Blue the optimal firepower whilst still being able to move a section to close down the assault on Red. Moving all three sections is a bayonet charge. These were chosen to represent extremes in the possible matrix of moves (see Table 1) to better understand the consequences of platoon move and fire. The diagram in the left hand column is option (b) and the diagram in the right hand column is option (h).

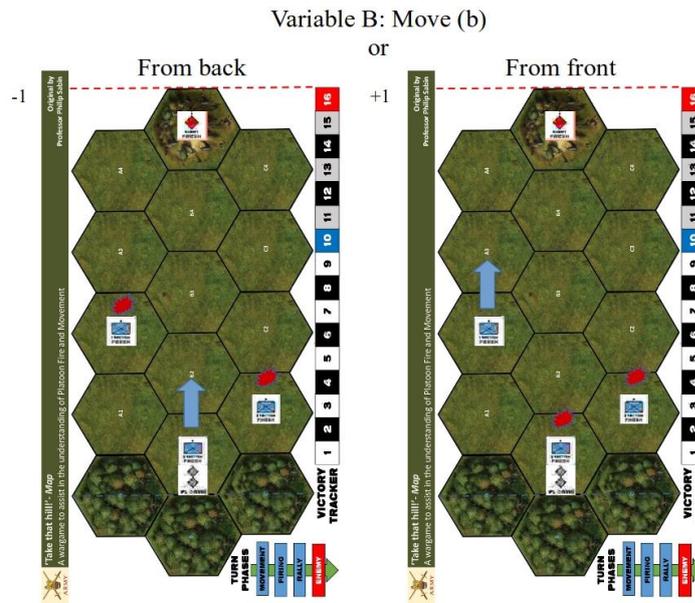


Figure 6: Variable B in the Experimental Design. The difference between moving from the back -1 or moving from the front +1.

Comments: Moving from the back maintains the overall attack momentum at the expense of speed to obtain the objective, whereas moving from the front attempts to close the enemy down in the quickest time.

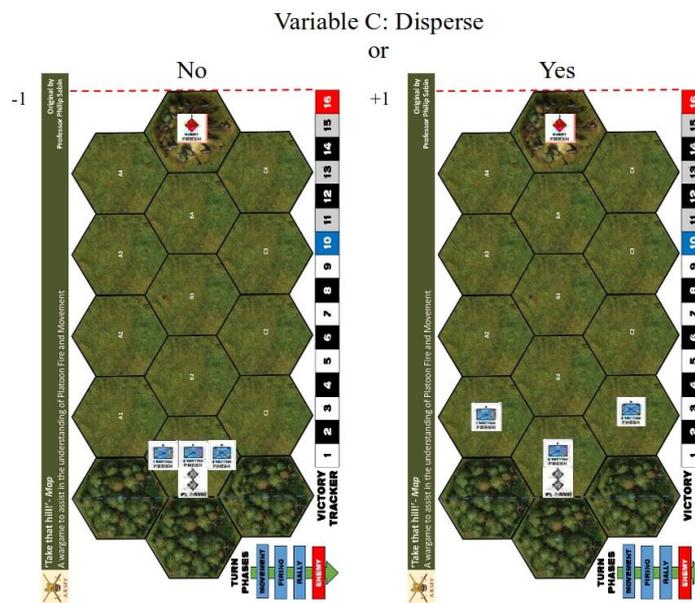


Figure 7: Variable C in the Experimental Design. The difference between keeping concentrated as a solid block -1 , or dispersed +1.

Comments: Concentration increases the chance of rallying in the presence of COMD, but increases the chance of suppressing fire from Red. Dispersion has the opposite effect.

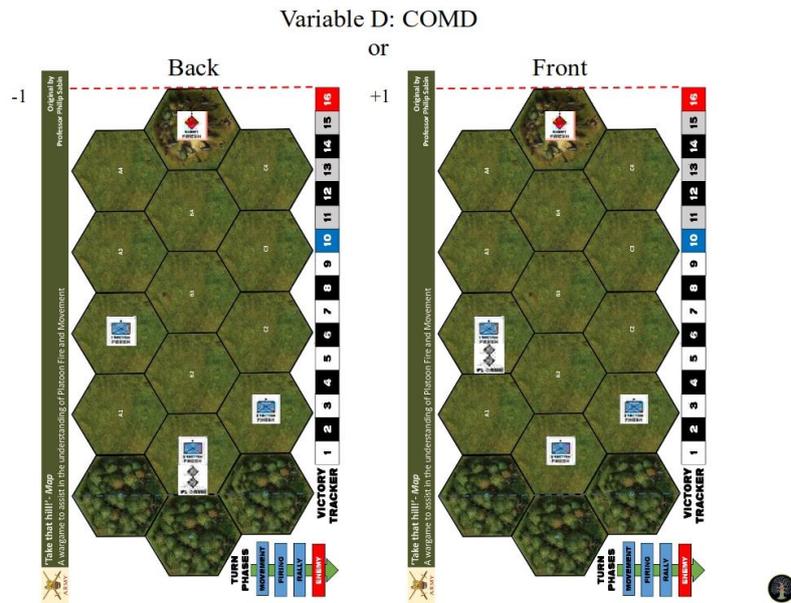


Figure 8: Variable D in the Experimental Design. The difference between COMD leading from the back -1, or COMD leads from the front +1.

Comments: The Platoon Commander (COMD) has the option of leading from the front to keep attack momentum up, or to lead from the back to rally stragglers and keep the attack moving overall.

Appendix 2 Statistical analysis of Red & Blue hits and Moves to the hill from the Experimental Design

Statistical methods such as scatter plots, best subsets regression and multiple linear regression analysis have been applied to the number of Red & Blue hits and Moves to the hill, as detailed in table 3 against the 4 key variables in the Experimental Design. The analysis is shown below for all three key metrics.

First, Red hits:

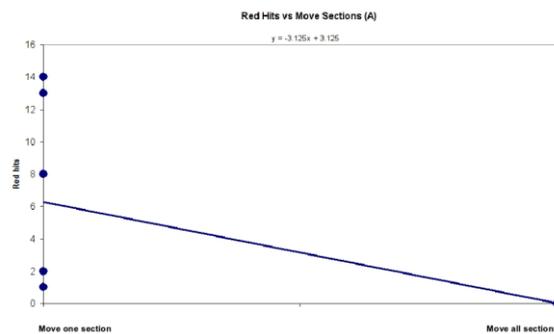


Figure 9: Red hits plotted against variable A, with moving 1 section, with the rest firing - 1 shown on the left, and all 3 sections moving +1 shown on the right.

There is a clear advantage in moving 1 section with the rest firing, compared to moving all sections, as might be expected when inflicting hits on Red. The strategy of moving all

sections corresponds to a bayonet charge which clearly cannot inflict hits on Red until Blue closes down on them.

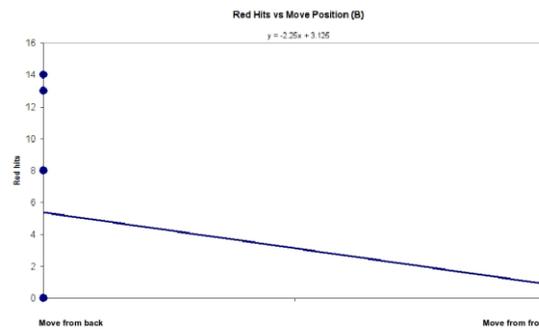


Figure 10: Red hits plotted against variable B, with moving sections from the back -1 shown on the left, and moving from the front +1 shown on the right.

There is a clear advantage in moving from the back, compared to moving from the front.

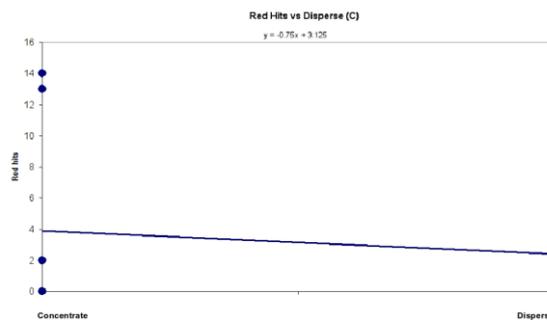


Figure 11: Red hits plotted against variable C, with keeping sections concentrated -1 shown on the left, and keeping sections dispersed +1 shown on the right.

There is an advantage in keeping concentrated, compared to keeping dispersed. This is surprising on first view, but it is believed this is due to the rules based on the proximity of the platoon commander (COMD) in automatically rallying sections in the same hex as COMD, compared to dispersing where there is a reduced chance of rallying as outlined in the introduction.

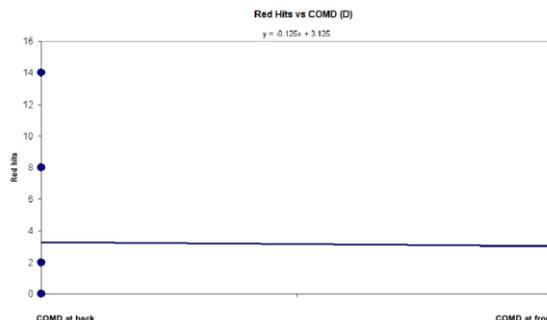


Figure 12: Red hits plotted against variable D, with COMD at the back -1 shown on the left, and COMD at the front +1 shown on the right.

There is no advantage in keeping the platoon commander (COMD) either at the front or the back.

The scatter plots indicate that for Red hits variables A, B & C are important, but not D. This is confirmed when using best subsets regression with Minitab ® 18.1.

Table 7: Results from best subsets analysis of Red hits vs key variables.

Variables	R ²	Adj. R ²	Cp	S	A	B	C	D
1	44.7	40.7	8.6	3.718	X			
1	23.2	17.7	16.6	4.381		X		
2	67.8	62.9	2.0	2.942	X	X		
2	47.2	39.1	9.7	3.767	X		X	
3	70.4	63.0	3.0	2.937	X	X	X	
3	67.9	59.9	4.0	3.059	X	X		X
4	70.5	59.7	5.0	3.064	X	X	X	X

The analysis indicated in bold highlights the 3 variables (A, B & C) in order of their importance which maximises both the degree of fit (R² and Adjusted R² with the adjusted value accounting for a reduction in degrees of freedom as variables are added to the model), Mallows Cp(which should be close to the number of variables selected), and minimum S (an estimate of the variability about the regression line), with smaller being better.

Based on this procedure, the regression model for Red hits is:

$$Red\ hits = 3.13 - 3.12 A - 2.25 B - 0.75 C \tag{1}$$

where the variables A, B & C have values -1 or +1 as defined in Table 2.

Second, Blue hits:

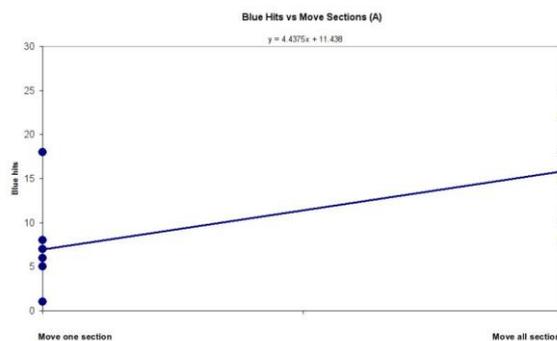


Figure 13: Blue hits plotted against variable A, with moving 1 section, with the rest firing -1 shown on the left, and all 3 sections moving +1 shown on the right.

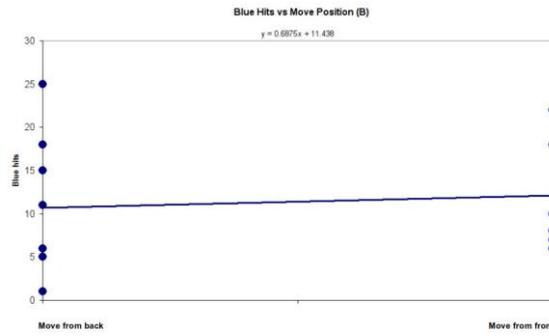


Figure 14: Blue hits plotted against variable B, with moving sections from the back -1 shown on the left, and moving from the front +1 shown on the right.

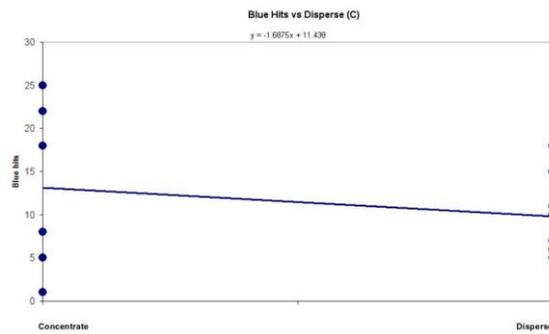


Figure 15: Blue hits plotted against variable C, with keeping sections concentrated -1 shown on the left, and keeping sections dispersed +1 shown on the right.

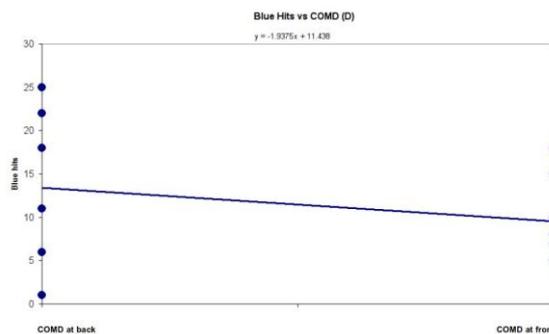


Figure 16: Blue hits plotted against variable D, with COMD at the back -1 shown on the left, and COMD at the front +1 shown on the right.

The scatter plots indicate that for Blue hits variables A, C & D are important, but not B. This is confirmed when using best subsets regression.

Table 8: Results from best subsets analysis of Blue hits vs key variables.

Variables	R ²	Adj. R ²	Cp	S	A	B	C	D
1	42.9	38.9	3.1	5.470	X			
1	8.2	1.6	12.2	6.938				X
2	51.1	43.6	2.9	5.254	X			X
2	49.1	41.3	3.4	5.359	X		X	
3	57.3	46.6	3.3	5.109	X		X	X
3	52.1	40.2	4.6	5.410	X	X		X
4	58.3	43.2	5.0	5.272	X	X	X	X

The analysis indicated in bold highlights the 3 variables (A, C & D) which maximises both the degree of fit (R² and Adjusted R² with the adjusted value accounting for a reduction in degrees of freedom as variables are added to the model), Mallows Cp (which should be close to the number of variables selected), and S is an estimate of the variability about the regression line, with smaller being better.

Based on this procedure, the regression model for Blue hits is:

$$Blue\ hits = 11.44 + 4.44 A - 1.69 C - 1.94 D \tag{2}$$

where the variables A, C & D have values -1 or +1 as defined in Table 2.

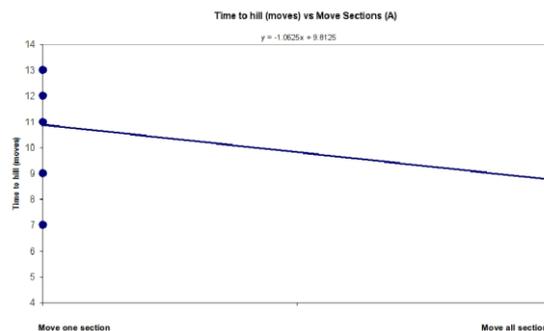


Figure 17: Time to hill plotted against variable A, with moving 1 section, with the rest firing -1 shown on the left, and all 3 sections moving +1 shown on the right.

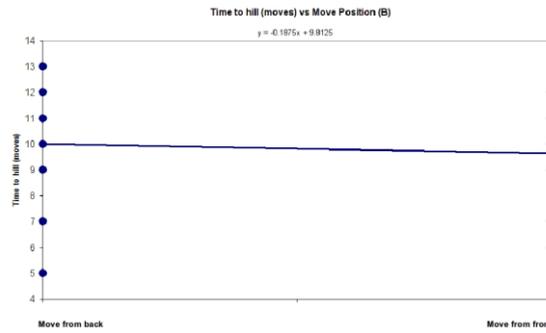


Figure 18: Time to hill plotted against variable B, with moving sections from the back -1] shown on the left, and moving from the front +1 shown on the right.

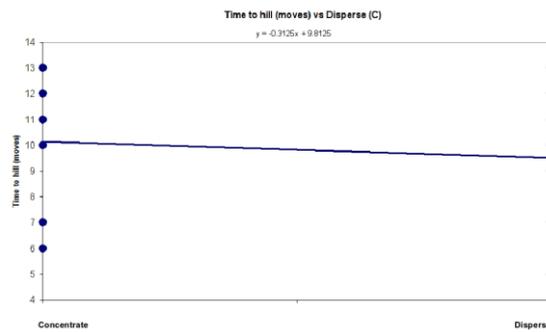


Figure 19: Time to hill plotted against variable C, with keeping sections concentrated -1 shown on the left, and keeping sections dispersed +1 shown on the right.

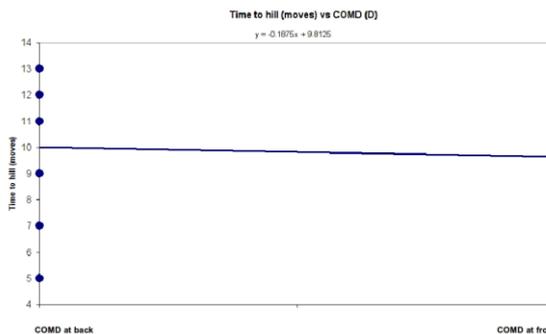


Figure 20: Time to hill plotted against variable D, with COMD at the back -1] shown on the left, and COMD at the front +1 shown on the right.

The scatter plots indicate that for Time to the hill variables A is important, but not B, C & D. This is confirmed when using best subsets regression.

Table 9: Results from best subsets analysis of Moves to hill vs key variables.

Variables	R ²	Adj. R ²	Cp	S	A	B	C	D
1	20.0	14.3	-0.6	2.274	X			
1	1.7	0.0	2.0	2.520			X	
2	21.7	9.7	1.2	2.334	X		X	
2	20.6	8.4	1.3	2.350	X			X
3	22.3	2.9	3.1	2.420	X		X	X
3	22.3	2.9	3.1	2.420	X	X	X	
4	22.9	0.0	5.0	2.517	X	X	X	X

The analysis indicated in bold highlights the single variables (A) which maximises both the degree of fit (R² and Adjusted R² with the adjusted value accounting for a reduction in degrees of freedom as variables are added to the model), Mallows Cp (which should be close to the number of variables selected), and S is an estimate of the variability about the regression line, with smaller being better.

Based on this procedure, the regression model for Moves to hill is:

$$\text{Moves to hill} = 9.81 - 1.063 A \quad (3)$$

where the variable A has the values -1 or +1 as defined in Table 2.

Appendix 3 Repeatability study on runs 1 & 13

The results from 5 repeats of runs 1 & 13 were:

Table 10: Repeatability from runs 1 and 13 to check consistency. The mean and standard deviation results are highlighted in bold.

	A	B	C	B	Red Hits	Blue Hits	Time to take hill
Run	Move sections	Move position	Disperse	COMD			
1	1 if possible	From back	No	Back	14	1	12
1	1 if possible	From back	No	Back	6	4	11
1	1 if possible	From back	No	Back	7	5	10
1	1 if possible	From back	No	Back	5	13	11
1	1 if possible	From back	No	Back	9	3	12
				Mean (Std Dev)	8.2 (3.4)	5.2 (4.6)	11.2 (0.8)
13	1 if possible	From back	Yes	Front	8	5	11
13	1 if possible	From back	Yes	Front	4	16	12

13	1 if possible	From back	Yes	Front	6	9	9
13	1 if possible	From back	Yes	Front	6	13	13
13	1 if possible	From back	Yes	Front	6	14	14
				Mean (Std Dev)	6.0 (1.4)	11.4 (4.4)	11.8 (1.9)

Under repeat conditions there is more spread in the results, but run 1 still outperforms run 13 on average, and at the 93% confidence level in a t-test regarding a reduction in Blue hits. Graphically the repeats show the following.

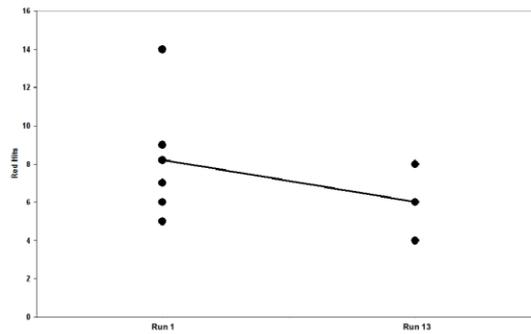


Figure 21: Red hits for Runs 1 & 13 showing an advantage to Run 1

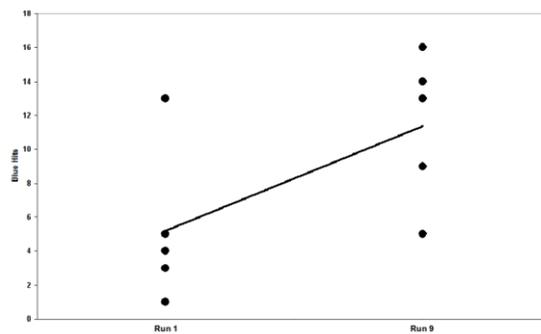


Figure 22: Blue hits for Runs 1 & 13 showing a distinct advantage to Run 1

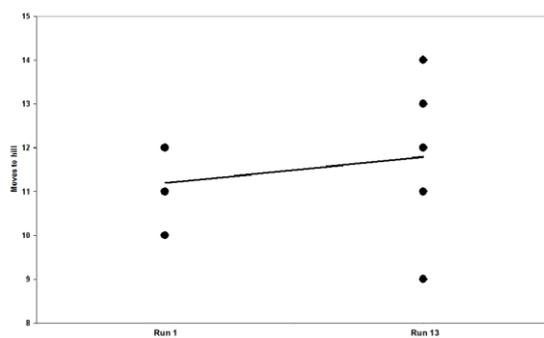


Figure 23: Moves to hill for Runs 1 & 13 showing an advantage to Run 1